DYNAMIC ANALYSIS OF THE GUIDING & SUSPENSION SYSTEM OF THE VEHICLES' REAR AXLE IN MULTI-BODY SYSTEMS CONCEPT

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Abstract: In this paper, the dynamic analysis of the suspension system for the rear axle of the motor vehicles is performed considering the guiding mechanism as well as the elastic and damping elements. The mathematical model was developed using the Newton - Euler formalism. The generalized coordinates in the axle suspension system are given for the initial position of the mechanism, their evolution during the motion being governed by a set of constraint kinematic equations. The simulation was performed with the MBS (Multi-Body Systems) software solution MSC.ADAMS, considering the passing over bumps dynamic regime. The analysis purpose was to determinate the influence of the springs stiffness on the vertical acceleration of the car body, which is the main factor for evaluating the vehicle comfort.

1. INTRODUCTION

In relative motion to car body, the rear axle of the cars are guided by spatial mechanisms, on which between axle (i.e. guided body) and car body a number of binary links are interposed. The links connections to axle and car body are made through compliant joints (bushings). Under the action of the forces in the suspension system, the bushings undergo elastic restricted linear and angular deformations, the "joint" having in fact six degrees of freedom (DOF). Usually, the modeling of the bushing element as spherical joint, with three DOF, is used; the structural systematization of the axle guiding linkage takes into consideration this assumption [1].

The increasingly growing demand for more comfortable cars imposes a new way for the dynamic analysis of the guiding axle linkages, with elaboration of models that are closer to the real mechanisms on the car. The complexity of the guiding mechanisms of the passenger cars makes it very difficult to handle the error in the case of simultaneous achievement of the entire model, therefore in the preliminary stages partial models are "experimented" (for the wheel suspension, and rear axle suspension respectively).

In this paper, a dynamic model for the rear axle suspension is presented, considering the guiding mechanism as well as the elastic and damping elements. The mathematical model was developed using the Newton - Euler formalism. The generalized coordinates in the axle suspension system are given for the initial position of the mechanism, their evolution during the motion being governed by a set of constraint kinematic equations.

The dynamic simulation was performed with the MBS (Multi-Body Systems) software MSC.ADAMS, considering the passing over bumps dynamic regime. The analysis was achieved for a time interval long enough to catch all relevant motions during the experiment.

2. THE DYNAMIC MODEL OF THE GUIDING & SUSPENSION SYSTEM

Traditionally, for the dynamic analysis of the suspension systems of the motor vehicles, the literature presents various linear models, neglecting the guiding linkages of the front and rear unsprung masses. Such models, with seven degrees of freedom, are shown in figure 1, as follows:

a. $q_1 = Z_0$, $q_2 = Z_1'$, $q_3 = Z_2'$, $q_4 = Z''$, $q_5 = \alpha$, $q_6 = \beta$, $q_7 = \beta''$;

b. $q_1 = Z_0$, $q_2 = Z_1'$, $q_3 = Z_2'$, $q_4 = Z_1''$, $q_5 = Z_2''$, $q_6 = \alpha$, $q_7 = \beta$;

c. $q_1 = Z_0$, $q_2 = Z'$, $q_3 = Z''$, $q_4 = \alpha$, $q_5 = \beta$, $q_6 = \beta'$, $q_7 = \beta''$;

where $Z_0(t)$ is vertical displacement of the sprung mass (i.e. the car body); $Z'(t)$ - vertical displacement of the front axle; $Z''(t)$ - vertical displacement of the rear axle; $\alpha(t)$ - pitch angle of the sprung mass; $\beta(t)$ - roll angle of the sprung mass; $\beta'(t)$ - roll angle of the front axle, $\beta''(t)$ - roll angle of the rear axle.

More complex models can be obtained by considering the geometrical restrictions (the guiding linkages of the wheels/axle) and the nonlinear characteristics of the elastic and damping elements (springs, dampers, rubber bumpers limiting the run, anti-roll bar, tires). Involving the system elasticity and the car body oscillations, the degree of freedom of the model is increasing, and it is practically impossible to analyze such models with classical methods / programs. Under these circumstances, it is necessary to use mechanical systems analysis software (MBS), which automatically formulate and solve the equations of motion by taking into consideration the geometric -elastic model of the mechanism, and the constraints in motion [2-4].

a.

b.

c.

Figure 1. Linear models for the suspension systems

In the last years, the author carried out a series of studies on the passenger car suspension systems, based on complex models achieved with the multi-body software ADAMS (Automatic Dynamic Analysis of the Mechanical Systems). ADAMS is the world's most widely used mechanical system simulation software. It enables users to produce virtual prototypes, realistically simulating the full-motion behavior of complex mechanical systems on their computers and quickly analyzing multiple design variations until an optimal design is achieved. This reduces the number of costly physical prototypes, improves design quality, and significantly reduces product development time [5-7].

The dynamic model of the axle suspension, shown in figure 2, is characterized as a constrained, multi-body, spatial mechanical system, in which kinematic elements (parts) are connected through compliant joints (bushings) and force elements such as nonlinear springs, dampers. The suspension system is modeled and analyzed in a global coordinate system (GCS), which is attached to ground (fixed body). For each moving body, a local
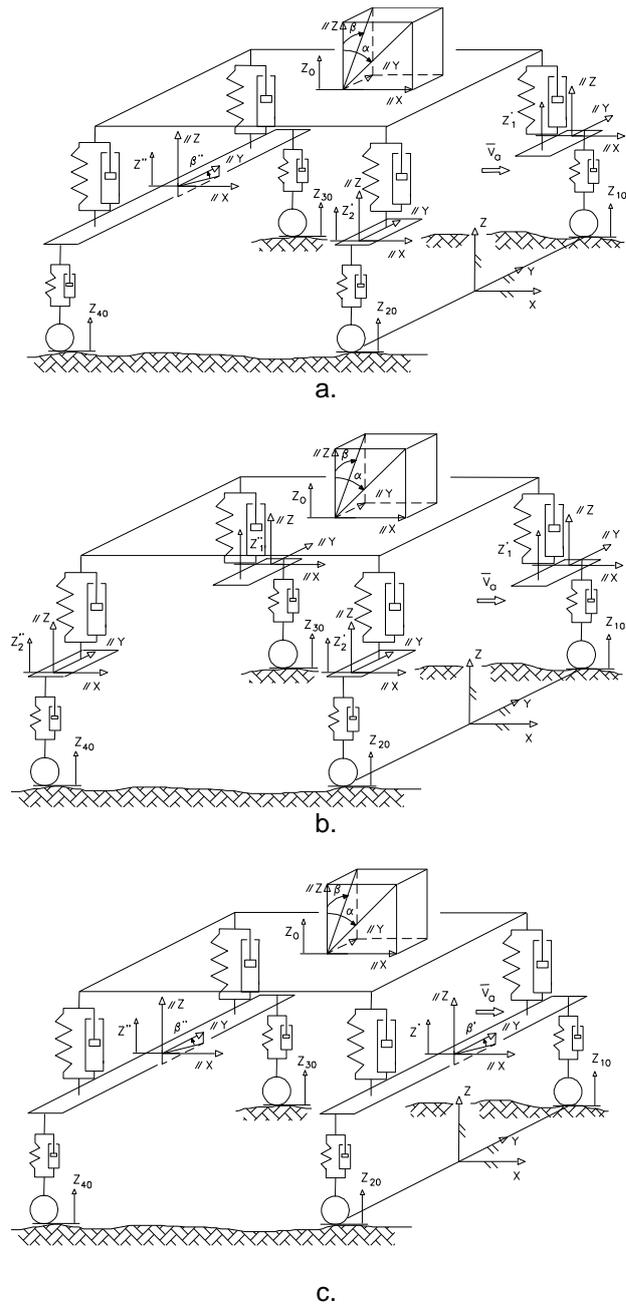
coordinate system (LCS) is defined, which is fixed in that body. The LCS position and orientation relative to GCS describe, in fact, the body motion.
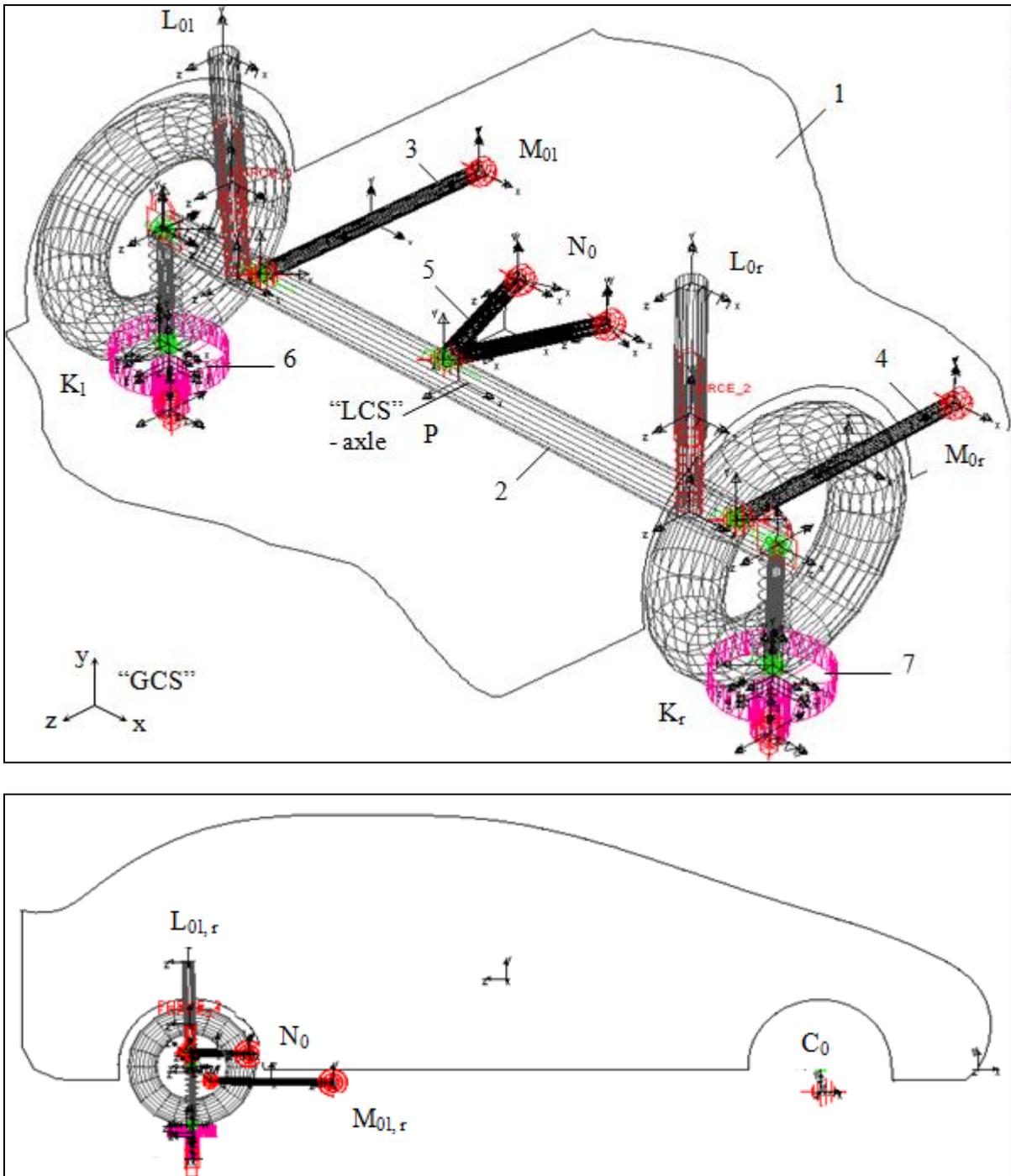

Figure 2. The virtual model of the guiding & suspension system of the rear axle (MSC.ADAMS)

For modeling rigid bodies (car body 1, rear axle 2, lower 3, 4 and upper 5 control arms of the guiding mechanism), ADAMS offers a set of solid geometries. These are three-dimensional objects with mass and inertia properties. The mass of the part, the center of mass position and the inertia properties are automatically calculated in relation to the part's geometry and the material properties. The elementary solid objects can be combined into more complex geometry by using boolean operations (union, subtract and intersection).

Constraints, which define how bodies are attached and how they are allowed to move relative to each other, can be idealized joints, which have a physical counterpart, and motions generators, which drive the model; each constraint removes a different number of DOF. To simplify the guiding linkage model, the connections of the upper and lower arms to car body and axle can be modeled as spherical joints, which allow the free rotation about a common point of one part with respect to another part.

For modeling the drivers (kinematic restrictions) that dictate the movement of the parts as functions of time (position, velocity or acceleration - linear or angular), joint motions can be used. In this paper, the suspension system is analyzed in the passing over bumps regime, therefore two additional parts (6, 7) are used to model the driving actuators. The roadway profile is modeled by driver constraints (motion generators), which are applied in the translational joints of the left & right actuators to ground.

In the lack of the front suspension, modeling a fictive joint between car body and ground ensures the car body equilibrium. Usually, the car body equilibrium can be made with a spherical joint placed in the longitudinal plan of vehicle. The location (C_0) of the spherical joint was obtained on the basis of double conjugate points' theory [8].

Elastic and damping elements of the suspension system represents forces acting between two parts (usually, car body and axle) over a distance and along a particular direction. The suspension spring is modeled as a double active (tension – compression) elastic element of translational nature, between car body and axle. The inputs for modeling the springs are: global coordinates of the points in which the springs are connected to adjacent parts; length at preload; constant spring stiffness or spring force vs. deflection characteristic.

The internal forces of elastic bumpers have transitory character, so that these elastic elements were modeled as translational springs with unilateral rigidity, which are active only when spring is in tension or in compression (using one-sided impact force).

The tire can be modeled as a three-dimensional Hertz model that contains a spring in parallel with a damper, one for each direction, between axle (rims) and ground. At the same time, ADAMS contains various predefined models for tires (Delft, Fiala, Smithers).

The degree of freedom (DOF) of the model is equal to the difference between the number of allowed part motions and the number of geometric & kinematic restrictions (Σr), DOF= $6 \cdot n - \Sigma r = 42 - 15 = 27$, as follows:

- generalized coordinates for 7 mobile parts (car body, rear axle, lower & upper control arms, driving actuators): $7 \times 6 = 42$;
- degrees of freedom restricted by constraints: spherical joint between car body and ground: $1 \times (-3) = -3$; translational joints between actuators and ground: $2 \times (-5) = -10$, motions generators (kinematic restrictions): $2 \times (-1) = -2$.

Considering the bushings modeled as spherical joints, the degree of freedom of the suspension system decreases, DOF = 7. Evidently, joining the triangular upper link to car body by two spherical joints ($N_{0'}$, $N_{0''}$), a revolute joint (N_0) is obtained. Consequently, the dynamic model has 7 independent generalized coordinates, namely: three rotations (φ_{1x}, φ_{1y}, φ_{1z}) for car body, the vertical position (Y_P) and the roll angle (φ_{2z}) for axle, the passive rotations of the lower control arms around their axes (φ_{3z}, φ_{4z}).

3. THE DYNAMIC EQUATIONS OF MOTION

The generalized coordinates are given for the initial position, their evolution during the simulation being governed by a set of constraint equations $F_{ij}(q_i, q_j)=0$, where $q_{i,j}$ represents the generalized coordinates column matrix of parts i, j. For the guiding mechanism shown in figure 2, there are the following geometric constraint equations:

- spherical joints (M_l, M_r, N) between control arms 3, 4, 5 and axle 2:

$$F_{32} = [r_P] + [M_{20}] \cdot [r_{Ml}]_2 - [r_{M0l}] - [M_{30}] \cdot [r_{Ms}]_3 = 0,$$
$$F_{42} = [r_P] + [M_{20}] \cdot [r_{Mr}]_2 - [r_{M0r}] - [M_{40}] \cdot [r_{Md}]_4 = 0, \qquad (1)$$
$$F_{52} = [r_P] + [M_{20}] \cdot [r_N]_2 - [r_{N0}] - [M_{50}] \cdot [r_N]_5 = 0,$$

where $[M_{i0}]$ represents the transformation matrix LCS → GCS for part 'i', $[r_P]$ – position vector of the axle origin, $[r_{M, N}]_2$ – position vectors of guiding axle points M_l, M_r, N in axle system, $[r_{M, N}]_{3-5}$ – position vectors of guiding points in local systems of the control arms;

- spherical (M_{0l}, M_{0r}) and revolute (N_0) joints between arms 3, 4, 5 and car body 1:

$$F_{31} = \begin{bmatrix} X_{M0l} \\ Y_{M0l} \\ Z_{M0l} \end{bmatrix} - \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = 0, \quad F_{41} = \begin{bmatrix} X_{M0r} \\ Y_{M0r} \\ Z_{M0r} \end{bmatrix} - \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = 0,$$

$$F_{51} = \begin{bmatrix} X_{N0} \\ Y_{N0} \\ Z_{N0} \\ \varphi_{5y} \\ \varphi_{5z} \end{bmatrix} - \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \\ \varphi_{1y} \\ \varphi_{1z} \end{bmatrix} - \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = 0; \qquad (2)$$

- spherical joint (C_0) between car body and ground:

$$F_{10} = \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = 0; \qquad (3)$$

- translational joints (K_l, K_r) between actuators 6, 7 and ground:

$$F_{60} = \begin{bmatrix} X_{Kl} \\ Z_{Kl} \\ \varphi_{6x} \\ \varphi_{6y} \\ \varphi_{6z} \end{bmatrix} - \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = 0, \quad F_{70} = \begin{bmatrix} X_{Kr} \\ Z_{Kr} \\ \varphi_{7x} \\ \varphi_{7y} \\ \varphi_{7z} \end{bmatrix} - \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} = 0. \qquad (4)$$

The geometric constants a_i, b_i, c_j, d_i, e_j, f_j can be obtained from the initial position of the suspension system. The constraint equations form a system of 23 scalar relations between the 30 generalized coordinates. Therefore, another 7 differential equations are necessary. These equations will be obtained using the Newton-Euler formalism as follows,

$$\begin{bmatrix} [m_i][0] \\ [0] [I_i] \end{bmatrix} \cdot \begin{bmatrix} [\ddot{r}_i] \\ [\ddot{\theta}_i] \end{bmatrix} = \begin{bmatrix} [R_i] \\ [M_{ri}] + [\tilde{\omega}_i] \cdot [I_i] \cdot [\omega_i] \end{bmatrix}_{i=1..7}, \qquad (5)$$

where: $[m_i]$ - mass diagonal matrix of part 'i'; $[I_i]$ - inertia tensor matrix; $[\ddot{\theta}_i]$, $[\ddot{r}_i]$ - angular and linear generalized accelerations; $[\omega_i]$ - angular velocity column matrix; $[\tilde{\omega}_i]$ - antisymmetrical matrix; $[R_i]$, $[M_i]$ - external resultant force/torque acting on part 'i'.

The resultant force and torque were determined from the equilibrium equations of the parts. As example, for car body, taking into account the elastic & damping forces (applied in L_{0l}, L_{0r}), the reaction forces & torques in the joints to lower/upper arms (M_{0l}, M_{0r}, N_0) respectively in the spherical joint to ground (C_0), and the mass G_1, the resultant force & torque will be:

$$[R_1]_1 = [F_{M0l}]_1 + [F_{M0r}]_1 + [F_{N0}]_1 + [F_{L0l}]_1 + [F_{L0r}]_1 + [F_{C0}]_1 + [M_{10}]^T \cdot [G_1], \qquad (6)$$
$$[M_1]_1 = [F_{M0l}]_1 \cdot [r_{M0l}]_1 + [F_{M0r}]_1 \cdot [r_{M0r}]_1 + [F_{N0}]_1 \cdot [r_{N0}] + [F_{L0l}]_1 \cdot [r_{L0l}]_1 + [F_{L0r}]_1 \cdot [r_{L0r}]_1 + [F_{C0}]_1 \cdot [r_{C0}]_1 + [M_{N0}]_1.$$

2.5

Explaining the equations (5) for each moving body (car body, axle, left & right lower control arms, upper control arm, left & right actuators), seven differential equations will be obtained, which along the constraint equations (1-4) determine the mixed system of the dynamic equations of motion for the considered guiding & suspension system.

4. RESULTS AND CONCLUSIONS

The dynamic analysis and simulation of the model was performed with the MBS software ADAMS. The experiment designed is one frequently carried by the automotive manufacturers, namely passing over bumps. The roadway profile was modelled by driver constraints that are applied to the driving actuators, as follows: left wheel pass over a roadway bump that has 80 mm height; right wheel runs on smooth surface (fig. 3). The analysis purpose was to determinate the influence of the springs stiffness - k [N/mm] - on the vertical acceleration of car body (fig. 4), which is the main factor (motion parameter) for evaluating the vehicle comfort.

Different other parameters (motion, force) can be obtained in a similar way, allowing fast and accurate evaluation of the dynamic behaviour. An important advantage of the simulation in virtual environment consists in the possibility of make virtual measurements in any point and area of the system, and for any parameter, without going through expensive prototype building and testing.
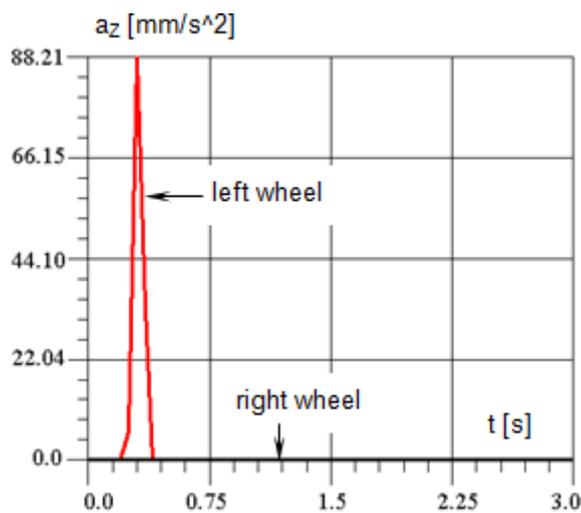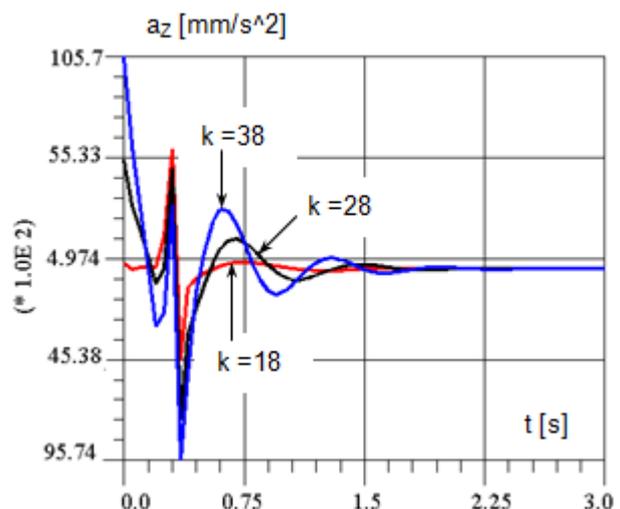

Figure 3. The roadway profile

Figure 4. The car body response

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